Solution to Problems ♡–7

Problem A: *Can you divide the three dimensional Euclidean space $\mathbb{R}^3$ into 2017 congruent disjoint pieces ?*

Answer: Yes. For $i = 0, 1, 2, 3, \ldots, 2016$ let

$$A_i = \Big\{ (x, y, z) \in \mathbb{R}^3 : \big(\exists k \in \mathbb{Z}\big)\big(2017k + i \leq x < 2017k + i + 1\big) \Big\}.$$

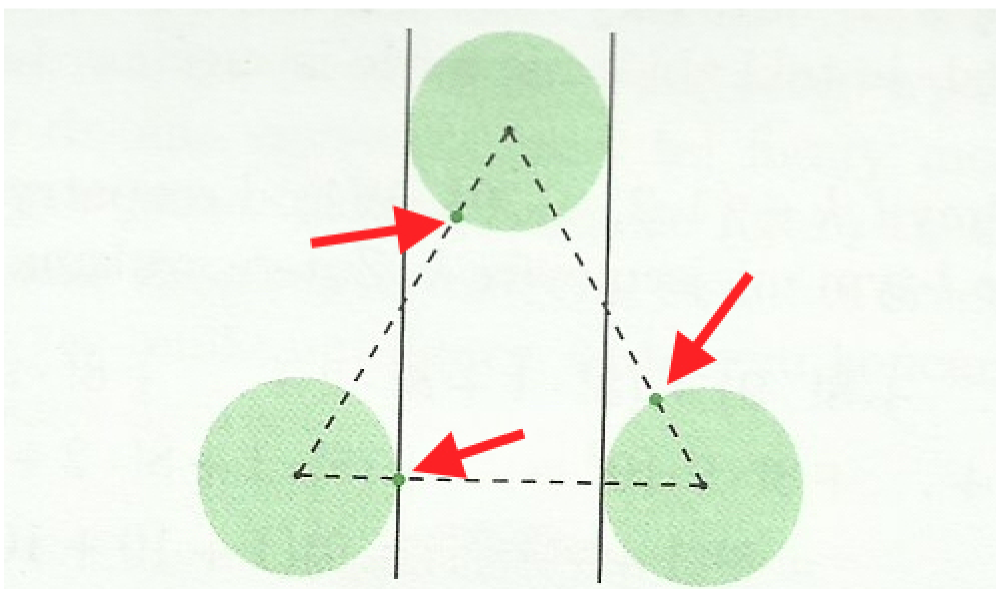
It should be clear that $\mathbb{R}^3 = \bigcup_{i=0}^{2016} A_i$, $A_i \cap A_j = \emptyset$ for distinct i, j and if $i < j$ then the translation by the vector $(j - i, 0, 0)$ maps the set A_i onto the set A_j.

Correct solution was received from :

(1) Cody Anderson

Problem B: *Does there exist a subset X of the plane with the property that the orthogonal projection of X onto any line is the union of two disjoint open line segments ?*

Answer: Yes, a set X with the required property can be obtained as the union of three *open* discs of radius 1 centered at the vertices of an equilateral triangle with side of length 4 and the set consisting of three points on the edges of the discs. [See the picture below; the arrows point to the three boundary points being added to the union of the three discs.]



No correct solution were received